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Note about static D1-brane in I-brane background

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Abstract: In this short note we will construct the static solutions on the world volume of D1-brane embedded in I-brane background.

Keywords: Intersecting branes models, D-branes.

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Contents

1. Introduction and summary

In the recent paper by N. Itzaki, D. Kutasov and N. Seiberg [1] the background that consists two stacks of fivebranes in type-IIB theory that intersect on $R^{1,1}$ was studied. Among many interesting results discovered there is the one that claims that there is an enhancement of the symmetry in the near horizon region of the I-brane. More precisely, it was shown that in the near horizon limit a combination of two transverse directions naturally combines with the world volume dimensions of I-brane where I-brane is defined classically as the intersection of two orthogonal stacks of NS5-branes. For detailed explanation of this result we recommend paper [1] for further reading.

In our recent paper [2] we have studied the time dependent dynamics of D1-brane probe in this background and we have given another evidence for an existence of the enhancement of the symmetry in the near horizon region. In this paper we will continue this study of the properties of I-brane background from the point of view of a D1-brane probe. We focus now on the static solutions on the world volume theory of D1-brane. We perform an analysis in the near horizon region where the exact static solution can be found. Following [2] we study this region from two different points of view. The first one is based on an existence of the additional symmetry on the world volume of D1-brane that allows us -together with the existence of the world volume stress energy tensor- to find exact static solutions. In the second approach we firstly perform the transformations used in [1] on the world volume of D1-brane. This transformation makes the symmetry enhancement manifest and the static solution can be easily found. It turns out that these two approaches give the same results.

On the other hand as will be clear from the nature of these solutions they cannot be valid for all points on the world volume of D1-brane since at some points these solutions do not obey the near horizon approximation. Even if this an unpleasant property of these solutions it is still possible to give them physical interpretation that is based on an observation that this D1-brane has many common with the AdS_2-brane studied in [3] and recently in [4]. In particular, in [4] the static configuration of AdS_2-brane in the background of N NS5-branes was studied. We will show, in the same way as in the paper [3, 4] that these D-branes cannot reach the world-volume of I-brane and hence cannot correspond to the well know situation when D-brane is stretched in the transverse directions to the world volume of NS5-branes [5, 6]. On the other hand we will argue that there exist solutions of the D-brane equations of motion that in the near horizon region corresponds to D1-brane that ends on the world volume of I-brane. We find these solutions when we will study the D1-brane equation of motion without imposing the gauge where the spatial coordinate on the world volume of D1-brane is equal to the spatial coordinate on the world volume of I-brane. Then we will calculate the components of the target space stress energy tensor for these configurations and we find that the D1-brane wraps spatial curve in the space transverse to I-brane. We will also show that in the special case when the number of NS5-branes in two orthogonal stacks is equal we can find solutions that correspond to D1-brane that ends on I-brane and extends in the whole transverse space. Unfortunately we were not able to find such a solution for general number of NS5-branes in two orthogonal stacks.

In summary, we again present some evidence for the existence of the enhancement symmetry in the near horizon region of I-brane. The most natural extension of this approach is to search for an exact CFT description of such a D1-brane configuration, following [7, 8]. It would be also nice to see whether these static solutions have some imprint on the dual little string theory [9]. Our goal is also to understood the super symmetric properties of given solution. In particular it would be nice to see whether they can be studied using general form of the calibration conditions [10, 11].

The organization of this paper is as follows. In the next section (2) we review the basic facts about D1-brane in I-brane background and we also present the form of the conserved charges determined in [2]. Then in section (2) we obtain the static solutions of D1-brane equation of motions that are valid in the near horizon limit. Solutions that can be interpreted as D1-brane that ends on I-brane will be given in section (4). Finally, in section (5) we will determine the components of the target space stress energy tensor and evaluate them on the solutions obtain in previous sections.

2. Review of basic facts about D1-brane in the background of I-brane

In this section we briefly review the basic facts about I-brane background studied recently in the work [1]. This background consists stack of k_1 NS5-branes extended in $(0, 1, 2, 3, 4, 5)$ direction and the set of k_2 NS5-branes extended in $(0, 1, 6, 7, 8, 9)$ directions. Let us define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5) \ , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) \ . \end{aligned} \tag{2.1}$$

We will consider the background where k_1 NS5-branes are localized at the point $\mathbf{z}=0$ and k_2 NS5-branes localized at the point $\mathbf{y}=0$. The supergravity background corresponding to this configuration takes the form [12]

$$\begin{aligned}
\Phi(\mathbf{z},\mathbf{y}) &= \Phi_1(\mathbf{z})+\Phi_2(\mathbf{y}) \ , \\
g_{\mu\nu} &= \eta_{\mu\nu} \ , \qquad \mu,\nu=0,1 \ , \\
g_{\alpha\beta} &= e^{2(\Phi_2-\Phi_2(\infty))}\delta_{\alpha\beta} \ , \qquad H_{\alpha\beta\gamma}=-\epsilon_{\alpha\beta\gamma\delta}\partial^\delta\Phi_2 \ , \qquad \alpha,\beta,\gamma,\delta=2,3,4,5 \\
g_{pq} &= e^{2(\Phi_1-\Phi_1(\infty))}\delta_{pq} \ , \qquad H_{pqr}=-\epsilon_{pqrs}\partial^s\Phi_1 \ , \qquad p,q,r,s=6,7,8,9 \ ,
\end{aligned} \tag{2.2}$$

where Φ on the first line means the dilaton and where

$$\begin{aligned}
e^{2(\Phi_1-\Phi_1(\infty))} &= H_1(\mathbf{z}) = 1+\frac{k_1 l_s^2}{|\mathbf{z}|^2} \ , \\
e^{2(\Phi_2-\Phi_2(\infty))} &= H_2(\mathbf{y}) = 1+\frac{k_2 l_s^2}{|\mathbf{y}|^2} \ .
\end{aligned} \tag{2.3}$$

We start with DBI action for D1-brane that moves in I-brane background

$$S=-\tau_1\int d^2x e^{-\Phi}\sqrt{-\det\mathbf{A}} \ , \tag{2.4}$$

where τ_1 is D1-brane tension. We restrict ourselves in this section to the gauge fixed form of the D1-brane action where the static gauge is defined as

$$X^0=x^0 \ , \qquad X^1=x^1 \ . \tag{2.5}$$

More general case will be studied in section (4). For the static gauge (2.5) the matrix $\mathbf{A}_{\mu\nu}$ in (2.4) takes the form

$$\mathbf{A}_{\mu\nu}=g_{\mu\nu}+g_{IJ}\partial_\mu X^I\partial_\nu X^J+b_{IJ}\partial_\mu X^I\partial_\nu X^J+\partial_\mu A_\nu-\partial_\nu A_\mu \ , \qquad I,J=2,\dots,9 \ . \tag{2.6}$$

where X^I parameterize the position of D1-brane in transverse space and where $A_\mu\ ,\mu\ ,\nu= 0,1$ are components of the world volume gauge field.

It turns out that in order to effectively analyze the properties of D1-brane in the I-brane background the knowledge of various world volume conserved quantities is useful. Natural example is the world volume stress energy tensor. It can be derived in various ways, for example one can introduce auxiliary world volume metric and perform the variation of the action with respect to it. It can be also determined by standard Noether procedure. This approach was used in our previous paper [2] and we got the result

$$\begin{aligned}
T^\mu_\nu = \tau_1 e^{-\Phi}\big(\delta^\mu_\nu - g_{IJ}\partial_\kappa X^J\left(\mathbf{A}^{-1}\right)^{\kappa\mu}_S\partial_\nu X^I - b_{IJ}\partial_\kappa X^J\left(\mathbf{A}^{-1}\right)^{\kappa\mu}_A\partial_\nu X^I - \\
-\partial_\nu A_\sigma\left(\mathbf{A}^{-1}\right)^{\sigma\mu}_A\big)\sqrt{-\det\mathbf{A}} \ ,
\end{aligned} \tag{2.7}$$

where

$$\left(\mathbf{A}^{-1}\right)^{\nu\mu}_S=\frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu}+\left(\mathbf{A}^{-1}\right)^{\mu\nu}\right) \ , \qquad \left(\mathbf{A}^{-1}\right)^{\nu\mu}_A=\frac{1}{2}\left(\left(\mathbf{A}^{-1}\right)^{\nu\mu}-\left(\mathbf{A}^{-1}\right)^{\mu\nu}\right) , \tag{2.8}$$

and where the components T^μ_ν obey the equation

$$\partial_\mu T^\mu_\nu = 0 \ . \tag{2.9}$$

In case when we have one dynamical variable the knowledge of T^μ_ν and the equation (2.9) are sufficient for determining the possible trajectory or static solution of given D1-brane. However it turns out that the dynamics of D1-brane in I-brane background is characterized by two dynamical modes. Even if we could in principle find the trajectories of given D1-brane by solving the equation of motion it is very difficult task.

The second possibility how to find possible D1-brane configurations is to search for another current that obeys the partial differential equation of the first order. As was shown in [2] such a current can be found in the near horizon limit of I-brane background. Since the near horizon limit is defined as

$$\frac{k_1 l_s^2}{|\mathbf{z}|^2} \gg 1 \ , \qquad \frac{k_2 l_s^2}{|\mathbf{y}|^2} \gg 1 \tag{2.10}$$

we can write

$$H_1 = \frac{\lambda_1}{|\mathbf{z}|^2} \ , \lambda_1 = k_1 l_s^2 \ , \qquad H_2 = \frac{\lambda_2}{|\mathbf{y}|^2} \ , \lambda_2 = k_2 l_s^2 \tag{2.11}$$

and consequently the action (2.4) takes the form

$$S = -\tau_1 \int d^2x \frac{1}{g_1 g_2 \sqrt{H_1 H_2}} \sqrt{-\det \mathbf{A}} \ , \tag{2.12}$$

where

$$\begin{aligned} \mathbf{A}_{\mu\nu} = & \ \eta_{\mu\nu} + H_1 \delta_{pr} \partial_\mu Z^p \partial_\nu Z^r + H_2 \delta_{\alpha\beta} \partial_\mu Y^\alpha \partial_\nu Y^\beta + \\ & + B_{pr} \partial_\mu Z^p \partial_\nu Z^r + B_{\alpha\beta} \partial_\mu Y^\alpha \partial_\nu Y^\beta + \partial_\mu A_\nu - \partial_\nu A_\mu \ , \end{aligned} \tag{2.13}$$

and where $g_1 = e^{\Phi_1(\infty)}$, $g_2 = e^{\Phi_2(\infty)}$. The form of the action (2.12) and the matrix (2.13) suggests that it is natural to consider following transformations

$$Z'^p(x') = \Gamma Z^p(x) \ , \qquad Y'^\alpha(x') = \Gamma^{-1} Y^\alpha(x) \ , \qquad A'_\mu(x') = A_\mu(x) \ , \qquad x'^\mu = x^\mu \ , \tag{2.14}$$

where Γ is a real parameter. It can be shown that this transformation implies, under some conditions, an existence of conserved charge. In fact, there exists a current j^μ_D related to the transformation (2.14) that has the form

$$\begin{aligned} j^\mu_D = & \ \tau_1 e^{-\Phi} \left(g_{pr} Z^p \partial_\nu Z^r \left(\mathbf{A}^{-1}\right)^{\nu\mu}_S + b_{pr} Z^p \partial_\nu Z^r \left(\mathbf{A}^{-1}\right)^{\nu\mu}_A \right) \sqrt{-\det \mathbf{A}} - \\ & - \tau_1 e^{-\Phi} \left(g_{\alpha\beta} Y^\alpha \partial_\nu Y^\beta \left(\mathbf{A}^{-1}\right)^{\nu\mu}_S + b_{\alpha\beta} Y^\alpha \partial_\nu Y^\beta \left(\mathbf{A}^{-1}\right)^{\nu\mu}_A \right) \sqrt{-\det \mathbf{A}} \end{aligned} \tag{2.15}$$

and that obeys the equation

$$\partial_\mu j^\mu_D = \tau_1 e^{-\Phi} \left(b_{pq} \partial_\mu Z^p \partial_\nu Z^q - b_{\alpha\beta} \partial_\mu Y^\alpha \partial_\nu Y^\beta \right) \left(\mathbf{A}^{-1}\right)^{\nu\mu} \sqrt{-\det \mathbf{A}} \ . \tag{2.16}$$

Note that the components g and b given above correspond to their near horizon limit. As follows from the right hand side of the equation (2.16), the conservation of j^μ_D is restored for homogenous modes or for modes that are spatial depend only since then the right side of the equation above vanishes thanks to the antisymmetry of b. The case of the pure time dependent modes was analyzed in our previous paper [2]. The static solutions will be studied in the next section.

3. Static solutions in the near horizon limit

In this section we will discuss the spatial dependent solution of D1-brane in I-brane background where all modes (except A_1) do not depend on x^0. Thanks to the manifest rotation symmetry SO(4) in the subspaces spanned by coordinates $\mathbf{z} = (z^6, z^7, z^8, z^9)$ and $\mathbf{y} = (y^2, y^3, y^4, y^5)$ we can reduce the problem to the study of the motion in two dimensional subspaces, namely we will presume that only following world volume modes are excited

$$z^6 = R\cos\theta \ , \qquad z^7 = R\sin\theta \ , \tag{3.1}$$

and

$$y^2 = S\cos\psi \ , \qquad y^3 = S\sin\psi \ . \tag{3.2}$$

There is also the gauge field A_1 present and we work in the gauge $A_0 = 0$.

Now using the fact that the action does not depend on angular modes explicitly it follows from the equation of motion that

$$\partial_1 \left[e^{-\Phi} g_{\theta\theta} \partial_1 \theta \left(\mathbf{A}^{-1}\right)_S^{11} \sqrt{-\det \mathbf{A}} \right] = 0 \tag{3.3}$$

that can be solved with $\theta = \theta_0 = \text{const}$. The same result holds for ψ as well and for that reason we will not discuss these modes at all.

Then for spatial dependent modes S, R and for nonzero A_1 we get following components of the matrix $\mathbf{A}$

$$\mathbf{A}_{00} = -1 \ , \qquad \mathbf{A}_{10} = F_{10} = -\dot{A}_1 = -\mathbf{A}_{01} \ , \qquad \mathbf{A}_{11} = 1 + H_1 R'^2 + H_2 S'^2 \ , \tag{3.4}$$

where $(\ldots)' \equiv \partial_1(\ldots)$. Then

$$\det \mathbf{A} = -(1 + H_1 R'^2 + H_2 S'^2) + (\partial_0 A_1)^2 \tag{3.5}$$

and also

$$\left(\mathbf{A}^{-1}\right) = \frac{1}{\det \mathbf{A}} \begin{pmatrix} 1 + H_1 R'^2 + H_2 S'^2 & \partial_0 A_1 \\ -\partial_0 A_1 & -1 \end{pmatrix} \ . \tag{3.6}$$

Using these results the components of the stress energy tensor (2.7) take the form

$$\begin{aligned} T_0^0 &= -\frac{\tau_1 e^{-\Phi}(1 + H_1 R'^2 + H_2 S'^2)}{\sqrt{-\det \mathbf{A}}} \ , \\ T_0^1 &= 0 \ , \qquad T_1^0 = -\frac{\tau_1 e^{-\Phi} \partial_1 A_1 \partial_0 A_1}{\sqrt{-\det \mathbf{A}}} \\ T_1^1 &= \tau_1 e^{-\Phi} \frac{(-1 + (\partial_0 A_1)^2)}{\sqrt{-\det \mathbf{A}}} \ . \end{aligned} \tag{3.7}$$

Firstly the equation (2.9) for $\nu = 0$ implies

$$\partial_0 T_0^0 = 0 \tag{3.8}$$

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that of course is trivially satisfied as T_0^0 does not depend on x^0 by construction. On the other hand the equation (2.9) for $\nu = 1$ gives

$$\partial_0 T_1^0 + \partial_1 T_1^1 \ , \tag{3.9}$$

where

$$\partial_0 T_1^0 = -\frac{\tau_1 e^{-\Phi} \partial_1 \partial_0 A_1 \partial_0 A_1}{\sqrt{-\det \mathbf{A}}} \ . \tag{3.10}$$

We see that the presence of nonzero $\partial_0 A_1$ makes the analysis more complicated. We will return to this problem later and now we restrict to the case when $\partial_0 A_1 = 0$. Then we also get $T_1^0 = 0$ and consequently

$$\partial_1 T_1^1 = 0 \Rightarrow T_1^1(x^1) = -\frac{\tau_1}{g_1 g_2 \sqrt{H_1 H_2} \sqrt{1 + H_1 R'^2 + H_2 S'^2}} = K \ , \tag{3.11}$$

where K is arbitrary constant. To find solution with nontrivial S and R we again restrict to the near horizon region where

$$H_1 = \frac{\lambda_1}{R^2} \ , \qquad H_2 = \frac{\lambda_2}{S^2} \ . \tag{3.12}$$

As we have argued in the previous section we can find in this region the dilatation current (2.15) that obeys the differential equation (2.16). Using the fact that the only dynamical modes are S, R we see that the right side of this equation vanishes. It is also easy to see that $j_D^0 = 0$ and the conservation of the j_D^μ reduces into

$$\partial_1 j_D^1 = 0 \ , \tag{3.13}$$

where

$$\begin{aligned} j_D^1 &= -\frac{\tau_1}{g_1 g_2 \sqrt{H_1 H_2}} \left(H_1 R R' - H_2 S S' \right) \sqrt{-\det \mathbf{A}} \\ &= -\frac{\tau_1 R S}{g_1 g_2 \sqrt{\lambda_1 \lambda_2}} \left(\frac{\lambda_1}{R^2} R R' - \frac{\lambda_2}{S^2} S S' \right) \frac{1}{\sqrt{1 + H_1 R'^2 + H_2 S'^2}} \ . \end{aligned} \tag{3.14}$$

It follows from the equation (3.13) that j_D^1 does not depend on x^1. Let us then denote its value as $j_D^1 \equiv D$. Using (3.11) we can express D as

$$D = K \left(\frac{\lambda_1 R'}{R} - \frac{\lambda_2 S'}{S} \right) \ . \tag{3.15}$$

By integration of this equation we get

$$\frac{D}{K} x^1 + D_0 = \ln R^{\lambda_1} - \ln S^{\lambda_2} \tag{3.16}$$

or equivalently

$$S = R^{\lambda_1/\lambda_2} e^{-\frac{1}{\lambda_2}(\frac{D}{K} x^1 + D_0)} \ . \tag{3.17}$$

To find the spatial dependent solution we use (3.15) to express S' as function of R, R' and S and then we insert it to the expression for T_1^1 (3.11)

$$-\frac{\tau_1 RS}{\sqrt{\lambda_1\lambda_2}g_1g_2}\frac{1}{\sqrt{1+\lambda_1\frac{R'^2}{R^2}+\frac{1}{\lambda_2}\left(\lambda_1\frac{R'}{R}-\frac{D}{K}\right)^2}}=K \tag{3.18}$$

that leads to the quadratic equation in the variable R'

$$\frac{\lambda_1^2}{\lambda}\frac{R'^2}{R^2}-2\frac{\lambda_1}{\lambda_2}\frac{D}{K}\frac{R'}{R}+1+\frac{D^2}{\lambda_2K^2}-\frac{\tau_1^2}{K^2\lambda_1\lambda_2(g_1g_2)^2}R^2S^2=0\ , \tag{3.19}$$

where $\frac{1}{\lambda}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$. This equation has two roots

$$R'=\frac{D\lambda R}{K\lambda_1\lambda_2}\pm\frac{\lambda R}{\lambda_1^2}\sqrt{\frac{\tau_1^2\lambda_1}{K^2(g_1g_2)^2\lambda_2\lambda}R^2S^2-\frac{\lambda_1^2}{\lambda}-\frac{\lambda_1}{\lambda_2}\frac{D^2}{K^2}}\ . \tag{3.20}$$

We will solve the equation (3.20) with the ansatz

$$R(x^1)=C(x^1)e^{\frac{D\lambda}{K\lambda_1\lambda_2}x^1}\ . \tag{3.21}$$

If we insert (3.21) into the equation (3.20) we obtain a differential equation for C in the form

$$C'=\pm\frac{\lambda}{\lambda_1^2}\sqrt{\frac{\lambda_1\tau_1^2e^{-2\frac{D_0}{\lambda_2}}}{K^2(g_1g_2)^2\lambda_2\lambda}C^{\frac{2\lambda_1}{\lambda}}-\left(\frac{\lambda_1^2}{\lambda}+\frac{\lambda_1}{\lambda_2}\frac{D^2}{K^2}\right)}\ , \tag{3.22}$$

where we have used the relation

$$R^2S^2=R^{2(\lambda_1+\lambda_2)/\lambda_2}e^{-\frac{2}{\lambda_2}(\frac{D}{K}x^1+D_0)}=C^{\frac{2\lambda_1}{\lambda}}e^{-2\frac{D_0}{\lambda_2}} \tag{3.23}$$

that follows from (3.17). After straightforward integration of the equation (3.22) we obtain following spatial dependence of R on x^1

$$R=e^{\frac{1}{\lambda_1+\lambda_2}(\frac{D}{K}x^1+D_0)}\left(\frac{\sqrt{\lambda_1\lambda_2}K(g_1g_2)\sqrt{1+\frac{1}{\lambda_1+\lambda_2}\frac{D^2}{K^2}}}{\tau_1}\right)^{\frac{\lambda}{\lambda_1}}\left(\frac{1}{\cos\frac{1}{\sqrt{\lambda}}\sqrt{1+\frac{D^2}{(\lambda_1+\lambda_2)K^2}}x^1}\right)^{\frac{\lambda}{\lambda_1}}. \tag{3.24}$$

Before we try to give a physical meaning of this result we determine this solution from a different point of view using the transformations proposed in [1]. Let us again start with the lagrangian density for D1-brane that is inserted in the near horizon region and again restrict to the case of the spatial dependent radial modes only

$$\mathcal{L}=-\frac{\tau_1}{g_1g_2}\sqrt{\frac{R}{\lambda_1}}\sqrt{\frac{S}{\lambda_2}}\sqrt{1+\frac{\lambda_1}{R^2}(\partial_1R)^2+\frac{\lambda_2}{S^2}(\partial_1S)^2}\ . \tag{3.25}$$

As the first step let us introduce two modes ϕ_1 and ϕ_2 as

$$R=e^{\frac{\phi_1}{\sqrt{\lambda_1}}}\ ,\qquad S=e^{\frac{\phi_2}{\sqrt{\lambda_2}}} \tag{3.26}$$

so that the lagrangian (3.25) takes the form

$$\mathcal{L} = -\frac{\tau_1 e^{(\frac{\phi_1}{\sqrt{\lambda_1}}+\frac{\phi_2}{\sqrt{\lambda_2}})}}{\sqrt{\lambda_1\lambda_2}g_1g_2}\sqrt{1+(\partial_1\phi)^2+(\partial_1\phi)^2}\ . \tag{3.27}$$

Then we introduce two modes ϕ, x^2 defined as

$$Q\phi = \frac{1}{\sqrt{\lambda_1}}\phi_1 + \frac{1}{\sqrt{\lambda_2}}\phi_2\ , \qquad Qx^2 = \frac{1}{\sqrt{\lambda_2}}\phi_1 - \frac{1}{\sqrt{\lambda_1}}\phi_2\ , \tag{3.28}$$

where

$$Q = \frac{1}{\sqrt{\lambda}}\ , \qquad \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}\ . \tag{3.29}$$

Now using the inverse transformations to (3.28)

$$\begin{aligned}\phi_1 &= \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1}x^2 + \sqrt{\lambda_2}\phi\right), \\ \phi_2 &= \frac{1}{\sqrt{\lambda_1+\lambda_2}}\left(\sqrt{\lambda_1}\phi - \sqrt{\lambda_2}x^2\right)\end{aligned} \tag{3.30}$$

the lagrangian (3.27) takes the form

$$\mathcal{L} = -\frac{\tau_1}{\sqrt{\lambda_1\lambda_2}g_1g_2}e^{Q\phi}\sqrt{1+(\partial_1x^2)^2+(\partial_1\phi)^2}\ . \tag{3.31}$$

Since this lagrangian does not explicitly depend on x^1 it follows that the quantity $T_1^1 = \frac{\delta\mathcal{L}}{\delta\partial_1x^2}\partial_1x^2 + \frac{\delta\mathcal{L}}{\delta\partial_1\phi}\partial_1\phi - \mathcal{L}$ obeys the equation $\partial_1T_1^1 = 0$ that again implies

$$-\frac{\tau_1}{\sqrt{\lambda_1\lambda_2}g_1g_2}e^{Q\phi}\frac{1}{\sqrt{1+(\partial_1x^2)^2+(\partial_1\phi)^2}} = K\ . \tag{3.32}$$

Also using the fact that the lagrangian does not depend on x^2 we immediately obtain following spatial independent quantity

$$\partial_1\left(\frac{\delta\mathcal{L}}{\delta\partial_1x^2}\right) = 0 \Rightarrow \frac{\delta\mathcal{L}}{\delta\partial_1x^2} = P_2 = \mathrm{const} \tag{3.33}$$

that allows us to write

$$\partial_1x^2 = \frac{P_2}{K} \tag{3.34}$$

and hence

$$x^2 = \frac{P_2}{K}x^1 + x_0^2\ . \tag{3.35}$$

If we insert (3.34) into to the equation (3.32) we obtain the differential equation for ϕ that has the form

$$\frac{d\phi}{\sqrt{\frac{\tau_1^2e^{2Q\phi}}{\lambda_1\lambda_2(g_1g_2)^2K^2} - \left(1+\frac{P_2^2}{K^2}\right)}} = \pm dx^1 \tag{3.36}$$

and that has the solution

$$e^{Q\phi} = \frac{\tau_1}{\sqrt{\lambda_1\lambda_2}g_1g_2K\sqrt{1+\frac{P_2^2}{K^2}}}\frac{1}{\cos Q\sqrt{1+\frac{P_2^2}{K^2}}x^1}\ . \tag{3.37}$$

If we perform an identification between P_2 and D in the form

$$P_2 = \frac{D}{\sqrt{\lambda_1 + \lambda_2}} \tag{3.38}$$

and use the relations (3.30) we can map the solution (3.37) to the solution (3.24). In other words, we have two equivalent descriptions of D1-brane in the near horizon region of I-brane. In the first one, that is valid for D1-brane in the near horizon region of the original background (2.2) the world volume action posses additional, scaling like symmetry. In the second one, based on the background introduced in [1] we get that the world volume theory is manifestly invariant in the direction x^2 and consequently the momentum P_2 is conserved. In both cases an enhancement of the symmetry in the near horizon region is observed.

Finally we return to the case when $\partial_0 A_1 \neq 0$. Firstly, recall that the equations of motion for A_μ take the form

$$\partial_\nu \left[e^{-\Phi} \left(\mathbf{A}^{-1}\right)^{\nu\mu}_A \sqrt{-\det \mathbf{A}} \right] = 0 \ , \tag{3.39}$$

or explicitly

$$\begin{aligned} \partial_1 \left[\frac{1}{g_1 g_2 \sqrt{H_1 H_2}} \frac{\partial_0 A_1}{\sqrt{1 + H_1 R^2 + H_2 S'^2 - (\partial_0 A_1)^2}} \right] &= 0 \ , \\ \partial_0 \left[\frac{1}{g_1 g_2 \sqrt{H_1 H_2}} \frac{\partial_0 A_1}{\sqrt{1 + H_1 R^2 + H_2 S'^2 - (\partial_0 A_1)^2}} \right] &= 0 \ . \end{aligned} \tag{3.40}$$

The second equation can be solved with $\partial_0^2 A_1 = 0$ while the first equation can be written as

$$\partial_1 [\partial_0 A_1 T^1_1] = \partial_1 \partial_0 A_1 T^1_1 + \partial_0 A_1 \partial_1 T^1_1 = 0 \tag{3.41}$$

that has again solution $\partial_1 A_1 = 0$ and $\partial_1 T^1_1 = 0$. This result together with the first equation in (3.40) implies that the quantity $\partial_0 A_1 T^1_1 \equiv -\pi$ is constant and we can write

$$\partial_0 A_1 = -\frac{\pi}{K} \ . \tag{3.42}$$

Then we obtain following form of the constant T^1_1

$$-\frac{\tau_1 R S}{g_1 g_2 \sqrt{\lambda_1 \lambda_2}} \frac{1}{\sqrt{1 + \frac{\lambda_1}{R^2} R'^2 + \frac{\lambda_2}{S^2} S'^2 - \frac{\pi^2}{K^2}}} = K \ , \tag{3.43}$$

where we have taken the near horizon limit in the end. Now we see that this equation has exactly the same form as the equation given in case when $\pi = 0$ with the small difference that the constant term under the square root is not 1 but $1 - \frac{\pi^2}{K^2}$. Then we could find the static solution exactly in the same way as above.

Now we return to the question of the physical interpretation of the spatial dependent solution given in (3.37) or in (3.24). We mean that it corresponds to an array of spikes that approach to the world volume of I-brane at distance $R_{\min}, S_{\min}$ at the points

$\left(1+\frac{D^2}{(\lambda_1+\lambda_2)K^2}\right)x_k = k\pi, k = 0,1,\dots$ and then they extend in R and S directions until they reach the distance where the near horizon approximation ceases to be valid. In some sense these solutions correspond to AdS_2-branes studied in paper [3] where the classical solution was valid for the whole AdS_3. This analysis was extended to the study of AdS_2 branes in the paper [4] and the results given there are similar to our solutions.

It is also important to stress that we cannot find smooth solution that describes D1-brane that terminates on the world volume of I-brane. In fact, the static gauge defined in the previous section is not suited for description of this configuration. In order to find such D1-brane static solution we should consider the general form of D1-brane effective action without where the static gauge is not imposed.

4. DBI action without static gauge presumption

In this section we present an alternative form of the embedding of the D1-brane in I-brane background. The starting point of our analysis is Dirac-Born-Infeld action for Dp-brane in a general background

$$S = -\tau_p \int d^{p+1}\sigma e^{-\Phi}\sqrt{-\det \mathbf{A}} \ , \qquad \mathbf{A}_{\mu\nu} = \gamma_{\mu\nu} + F_{\mu\nu} \ , \tag{4.1}$$

where τ_p is Dp-brane tension, $\Phi(X)$ is dilaton and where $\gamma_{\mu\nu}$, $\mu,\nu = 0,\dots,p$ is embedding of the metric to the world volume of Dp-brane

$$\gamma_{\mu\nu} = g_{MN}\partial_\mu X^M \partial_\nu X^N \ , \qquad M,N = 0,\dots,9 \ . \tag{4.2}$$

In (4.1) the form $F_{\mu\nu}$ is defined as

$$F_{\mu\nu} = b_{MN}\partial_\mu X^M \partial_\nu X^N + \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{4.3}$$

Note also that we label the world volume coordinates of Dp-brane with σ^μ , $\mu = 0,\dots,p$.

The equations of motion for X^K can be easily determined from (4.1) and take the form

$$\begin{aligned}
&\partial_K[e^{-\Phi}]\sqrt{-\det \mathbf{A}} + \\
&+\frac{1}{2}e^{-\Phi}\left(\partial_K g_{MN}\partial_\mu X^M\partial_\nu X^N + \partial_K b_{MN}\partial_\mu X^M\partial_\nu X^N\right)\left(\mathbf{A}^{-1}\right)^{\nu\mu}\sqrt{-\det \mathbf{A}} - \\
&-\partial_\mu\left[e^{-\Phi} g_{KM}\partial_\nu X^M\left(\mathbf{A}^{-1}\right)^{\nu\mu}_S\sqrt{-\det \mathbf{A}}\right] - \partial_\mu\left[e^{-\Phi} b_{KM}\partial_\nu X^M\left(\mathbf{A}^{-1}\right)^{\nu\mu}_A\sqrt{-\det \mathbf{A}}\right] = 0 \ .
\end{aligned} \tag{4.4}$$

Finally, we should also determine the equation of motion for the gauge field A_μ:

$$\partial_\nu\left[e^{-\Phi}\left(\mathbf{A}^{-1}\right)^{\nu\mu}_A\sqrt{-\det \mathbf{A}}\right] = 0 \ . \tag{4.5}$$

Using these equations of motion we present an alternative solutions corresponding to the static D1-brane in I-brane background. First of all we presume that all modes (except A_1, where we however demand that $\partial_0^2 A_1 = 0$) depend on σ^1 only. Since we consider time independent solution it is natural to fix

$$X^0 = \sigma^0 \ . \tag{4.6}$$

Then using an antisymmetry of b_{MN} it is clear that its embedding to the world volume of D1-brane is zero and consequently the matrix $\mathbf{A}_{\mu\nu}$ takes the form

$$\mathbf{A} = \begin{pmatrix} -1 & \partial_0 A_1 \\ -\partial_0 A_1 & g_{IJ}\partial_1 X^I \partial_1 X^J \end{pmatrix} \tag{4.7}$$

hence the determinant and the inverse matrix are equal to

$$\begin{aligned} \det \mathbf{A} &= -g_{IJ}\partial_1 X^I \partial_1 X^J + (\partial_0 A_1)^2 \ , & (4.8)\\ \left(\mathbf{A}^{-1}\right) &= \frac{1}{\det \mathbf{A}} \begin{pmatrix} g_{IJ}\partial_1 X^I \partial_1 X^J & -\partial_0 A_1 \\ \partial_0 A_1 & -1 \end{pmatrix} , & (4.9) \end{aligned}$$

where $I, J = 1, \dots, 9$. Then the equation of motion (4.5) implies

$$\begin{aligned} \partial_0 \left[e^{-\Phi} \left(\mathbf{A}^{-1}\right)^{10}_A \sqrt{-\det \mathbf{A}} \right] &= -\partial_0 \left[\frac{1}{\sqrt{H_1 H_2} g_1 g_2} \frac{\partial_0 A_1}{\sqrt{-\det \mathbf{A}}} \right] = 0 \\ \partial_1 \left[e^{-\Phi} \left(\mathbf{A}^{-1}\right)^{01}_A \sqrt{-\det \mathbf{A}} \right] &= \partial_1 \left[\frac{1}{\sqrt{H_1 H_2} g_1 g_2} \frac{\partial_0 A_1}{\sqrt{-\det \mathbf{A}}} \right] = 0 \ , \end{aligned} \tag{4.10}$$

The first equation is satisfied if we presume that $\partial_0 A_1$ does not depend on time. The second one implies that $\pi \equiv \frac{1}{\sqrt{H_1 H_2} g_1 g_2} \frac{\partial_0 A_1}{\sqrt{-\det \mathbf{A}}}$ does not depend on σ^1. In summary, we will solve these equations with $\pi = const$. For simplicity of resulting formulas we take $\pi = 0$.

As in previous section we use the manifest rotation symmetry SO(4) in the subspaces spanned by coordinates $\mathbf{z} = (z^6, z^7, z^8, z^9)$ and $\mathbf{y} = (y^2, y^3, y^4, y^5)$ so that we presume that only following world volume modes are excited

$$z^6 = R\cos\theta \ , \qquad z^7 = R\sin\theta \ , \tag{4.11}$$

and

$$y^2 = S\cos\psi \ , \qquad y^3 = S\sin\psi \ . \tag{4.12}$$

Since the metric does not depend on θ, ψ the equation of motions for them reduce into

$$\begin{aligned} \partial_1 \left[e^{-\Phi} g_{\theta\theta} \partial_1 \theta \left(\mathbf{A}^{-1}\right)^{11} \sqrt{-\det \mathbf{A}} \right] &= 0 \ , \\ \partial_1 \left[e^{-\Phi} g_{\psi\psi} \partial_1 \psi \left(\mathbf{A}^{-1}\right)^{11} \sqrt{-\det \mathbf{A}} \right] &= 0 \ , \end{aligned} \tag{4.13}$$

that can be solved with the ansatz

$$\psi = \psi_0 = \text{const} \ , \qquad \theta = \theta_0 = \text{const} \ . \tag{4.14}$$

Recall that there exists the mode X^1 on the world volume of D1-brane that parameterizes its position along the world volume of I-brane. This mode obeys the equation of motion

$$\partial_1 \left[e^{-\Phi} g_{11} \partial_1 X^1 \left(\mathbf{A}^{-1}\right)^{11}_S \sqrt{-\det \mathbf{A}} \right] = 0 \ . \tag{4.15}$$

If we denote $X^1(\sigma^1) = f$ then the equation above implies

$$\begin{aligned} &\frac{1}{g_1 g_2 \sqrt{H_1}\sqrt{H_2}} \frac{f'}{\sqrt{H_1(\partial_1 R)^2 + H_2(\partial S)^2 + (\partial_1 f)^2}} = K \Rightarrow \\ &\qquad \Rightarrow (\partial_1 f)^2 = \frac{H_1(\partial_1 R)^2 + H_2(\partial_1 S)^2}{\frac{1}{(g_1 g_2)^2 K^2 H_1 H_2} - 1} \ . \end{aligned} \tag{4.16}$$

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From the last equation we get following condition

$$\frac{1}{K^2(g_1g_2)^2} > H_1H_2 \ . \tag{4.17}$$

This condition implies that for $K \neq 0$ R, S cannot be equal to zero. In other words for $K \neq 0$ we cannot find solution that corresponds to the D1-brane that ends on the world volume of a I-brane. We have seen the similar result in the previous section. Then in order to find D1-brane that ends on I-brane we should take $K = 0$ that however implies $\partial_1 X^1 = 0$ and hence $X^1 = x_0^1$.

As we could expected the problem reduced to the study of the equation of motion for S, R. However in general case we are not able to proceed further without an existence of an additional symmetry. On the other hand we can gain some information about the solution that describes D1-brane that ends on I-brane if we restrict to the near horizon limit where we can either perform the transformation introduced in [1] or use the scaling like symmetry found in [2]. Since these two approaches are equivalent we restrict ourselves in this section to the first possibility. We again start with the lagrangian density of D1-brane in the near horizon region of I-brane

$$\mathcal{L} = -\frac{\tau_1}{g_1g_2}\sqrt{\frac{R^2}{\lambda_1}}\sqrt{\frac{S^2}{\lambda_2}}\sqrt{\frac{\lambda_1}{R^2}(\partial_1 R)^2 + \frac{\lambda_2}{S^2}(\partial_1 S)^2} \ . \tag{4.18}$$

Observe that this lagrangian density differs from the lagrangian density given in (3.25) in the fact that the factor 1 is absent in (4.18). Now we introduce two modes ϕ_1 and ϕ_2 defined as

$$R = e^{\frac{\phi_1}{\sqrt{\lambda_1}}} \ , \qquad S = e^{\frac{\phi_2}{\sqrt{\lambda_2}}} \tag{4.19}$$

and hence the lagrangian (4.18) takes the form

$$\mathcal{L} = -\frac{\tau_1 e^{(\frac{\phi_1}{\sqrt{\lambda_1}} + \frac{\phi_2}{\sqrt{\lambda_2}})}}{\sqrt{\lambda_1\lambda_2}g_1g_2}\sqrt{(\partial_1\phi)^2 + (\partial_1\phi)^2} \ . \tag{4.20}$$

Again using the transformations (3.30) we can rewrite the lagrangian density (4.20) into the more symmetric form

$$\mathcal{L} = -\frac{\tau_1}{\sqrt{\lambda_1\lambda_2}g_1g_2}e^{Q\phi}\sqrt{(\partial_1 x^2)^2 + (\partial_1\phi)^2} \ . \tag{4.21}$$

Since we did not fix the world volume coordinate σ^1 to be equal to some target space one it turns out that the spatial component of the world volume stress energy tensor T_1^1 vanishes identically. For that reason we should directly solve the equations of motion for x^2 and ϕ that follow (4.21). Firstly, using the fact that the lagrangian does not depend on x^2 we immediately obtain following spatial independent quantity

$$\partial_1\left(\frac{\delta\mathcal{L}}{\delta\partial_1 x^2}\right) = 0 \Rightarrow \frac{\delta\mathcal{L}}{\delta\partial_1 x^2} = \frac{\tau_1 e^{Q\phi}}{\sqrt{\lambda_1\lambda_2}g_1g_2}\frac{\partial_1 x^2}{\sqrt{(\partial_1 x^2)^2 + (\partial_1\phi)^2}} = P_2 \ . \tag{4.22}$$

From this equation we can also express $\partial_1 x^2$ as

$$(\partial_1 x^2)^2 = \frac{P_2^2(\partial_1\phi)^2}{\frac{\tau_1^2}{\lambda_1\lambda_2(g_1g_2)^2}e^{2Q\phi} - P_2^2} \ . \tag{4.23}$$

We are interested in the D1-brane that reaches the world volume of I-brane that occurs in the limit $\phi \to -\infty$. On the other hand the equation above implies the lower bound on the value of ϕ in the form

$$\frac{\tau_1^2}{\lambda_1\lambda_2(g_1g_2)^2}e^{2Q\phi} - P_2^2 > 0 \tag{4.24}$$

that can be obeyed in the limit $\phi \to -\infty$ in case when $P_2 = 0$ that however also implies $\partial_1 x^2 = 0$.

Then the equation of motion for ϕ takes the form

$$-\frac{\tau_1 Q}{\sqrt{\lambda_1\lambda_2}g_1g_2}e^{Q\phi}\sqrt{(\partial_1 x^2)^2 + (\partial_1\phi)^2} + \partial_1\left[\frac{\tau_1 e^{Q\phi}}{\sqrt{\lambda_1\lambda_2}(g_1g_2)}\frac{\partial_1\phi}{\sqrt{(\partial_1 x^2) + (\partial_1\phi)^2}}\right] = 0 \ . \tag{4.25}$$

It is easy to see that the equation of motion (4.25) can be solved with the ansatz (for $\partial_1 x^2 = 0$)

$$\phi(\sigma^1) = f(\sigma^1) \ , \tag{4.26}$$

where f is any smooth function. Recall that in the R, S variables this solution takes the form

$$R = e^{\sqrt{\frac{\lambda_2}{\lambda_1(\lambda_1+\lambda_2)}}f(\sigma^1)} \ , \qquad S = e^{\sqrt{\frac{\lambda_1}{\lambda_2(\lambda_1+\lambda_2)}}f(\sigma^1)} \ . \tag{4.27}$$

In order to gain better physical meaning of given solution we should calculate quantities that do not depend on the form of the function of f. Examples of such quantities are space time stress energy tensor or the currents corresponding to the coupling of D1-brane to Ramond-Ramond two form of to NS two form. We restrict ourselves to the calculation of the space time stress energy tensor that will be performed in section (5). Before we do this we would like to show that there exists the solution that is valid for all R, S. This solution can be found in case when $\lambda_1 = \lambda_2$ as we will see in the next subsection.

4.1 The case $\lambda_1 = \lambda_2 = \lambda$

We start with the case $\lambda_1 = \lambda_2 = \lambda$. We would like to show that in this case the DBI equation of motion have the solution

$$R = \sqrt{\lambda}g(\sigma^1) \ , \qquad S = \sqrt{\lambda}g(\sigma^1) \ , \tag{4.28}$$

for any smooth function $g(\sigma^1)$. As a justification for the choose of this ansatz note that the action is invariant under exchange R, S. For this ansatz we easily get

$$H_1 = H_2 = H \ , \qquad H_1(\partial_1 R)^2 + H_2(\partial_2 S) = 2\lambda H g'^2 \ , \tag{4.29}$$

where $H = 1 + \frac{1}{g^2}$. For simplicity we again restrict to the case of $\pi = 0$. It can be shown however that the same solution exists for $\pi \neq 0$ as well. Now if we insert the ansatz (4.28) and use (4.29) to the equation of motion for R we get

$$\begin{aligned}
&\partial_R[e^{-\Phi}]\sqrt{-\det\mathbf{A}}+\frac{1}{2}e^{-\Phi}\partial_R g_{RR}(\partial_1 R)^2\left(\mathbf{A}^{-1}\right)^{11}\ \sqrt{-\det\mathbf{A}}- \\
&\qquad -\partial_1\left[e^{-\Phi}g_{RR}\partial_1 R\left(\mathbf{A}^{-1}\right)^{11}\sqrt{-\det\mathbf{A}}\right]=\frac{1}{\sqrt{2}H^{3/2}}\frac{\lambda^{3/2}g'}{R^3}-\frac{\lambda^{3/2}g'}{\sqrt{2}R^3H^{3/2}} \\
&\qquad =0
\end{aligned} \tag{4.30}$$

that shows that (4.28) solves the equation of motion for R. Due to the symmetry between S and R it is clear that the equation of motion for S is obeyed as well.

In summary, in case of when the number of NS5-branes in two orthogonal stacks is the same we were able to find the static configuration corresponding to D1-brane extended in the space transverse to I-brane world volume. Since however this solution is given in terms of any function $g(\sigma^1)$ its interpretation is not completely clear. In order to gain better insight to its physical properties we will calculate the space time stress energy tensor for this configuration.

5. Space time stress energy tensor

In this section we determine components of the space time stress energy tensor and evaluate them on the solutions determined in the previous section. To begin with we again write the action as

$$S=-\tau_1\int d^{10}xd^2\sigma e^{-\Phi}\sqrt{-\det\mathbf{A}}\delta^{(10)}(X^M(\sigma)-x^M)\ . \tag{5.1}$$

Since the space time stress energy tensor is defined as

$$T_{MN}(x)=-\frac{2}{\sqrt{-g(x)}}\frac{\delta S}{\delta g^{MN}(x)} \tag{5.2}$$

we get

$$T_{MN}=-\frac{\tau_1}{\sqrt{-g(x)}}\int d^2\sigma e^{-\Phi}g_{MK}\partial_\mu X^K g_{NL}\partial_\nu X^L\left(\mathbf{A}^{-1}\right)^{\nu\mu}\sqrt{-\det\mathbf{A}}\delta^{(10)}(X^M(\sigma)-x^M)\ . \tag{5.3}$$

In our case we have $X^0=\sigma^0$. Then the integral over σ^0 can be easily performed and it is equal to one. Then for the spatial dependent ansatz and for $\partial_0 A_1=0$ we get

$$\begin{aligned}
T_{00}&=\frac{\tau_1}{\sqrt{-\det g(x)}}\int d\sigma^1\frac{\sqrt{H_1(\partial_1 R')^2+H_2(\partial_1 S)^2}}{g_1g_2\sqrt{H_1}\sqrt{H_2}}\delta(R(\sigma^1)-u)\delta(S(\sigma^1)-v)\ , \\
T_{uu}&=-\frac{\tau_1}{\sqrt{-\det g(x)}}\times \\
&\quad\times\int d\sigma^1\frac{H_1^2(\partial_1 R)^2}{g_1g_2\sqrt{H_1}\sqrt{H_2}\sqrt{H_1(\partial_1 R')^2+H_2(\partial_1 S)^2}}\delta(R(\sigma^1)-u)\delta(S(\sigma^1)-v)\ , \\
T_{vv}&=-\frac{\tau_1}{\sqrt{-\det g(x)}}\times \\
&\quad\times\int d\sigma^1\frac{H_2^2(\partial_1 S)^2}{g_1g_2\sqrt{H_1}\sqrt{H_2}\sqrt{H_1(\partial_1 R')^2+H_2(\partial_1 S)^2}}\delta(R(\sigma^1)-u)\delta(S(\sigma^1)-v)\ ,
\end{aligned}$$

$$T_{uv} = -\frac{\tau_1}{\sqrt{-\det g(x)}} \times$$
$$\times \int d\sigma^1 \frac{H_1 H_2 \partial_1 R \partial_1 S}{g_1 g_2 \sqrt{H_1}\sqrt{H_2}\sqrt{H_1(\partial_1 R')^2 + H_2(\partial_1 S)^2}} \delta(R(\sigma^1) - u)\delta(S(\sigma^1) - v) \ , \quad (5.4)$$

where we have omitted the delta functions that express that the components of the stress energy tensor are localized at fixed values x_0^1, θ_0, ψ_0. For the solution $R = g(\sigma^1)$, $S = g(\sigma^1)$ we get following result

$$\begin{aligned}
T_{00}(u,v) &= \frac{1}{\sqrt{-\det g(u,v)}} \frac{\tau_1\sqrt{2}}{g_1 g_2} \frac{1}{\sqrt{1+\frac{\lambda}{u^2}}} \delta(u-v) \ , \\
T_{uu}(u,v) &= -\frac{1}{\sqrt{-\det g(u,v)}} \frac{\tau_1}{g_1 g_2 \sqrt{2}} \sqrt{1+\frac{\lambda}{u^2}} \delta(u-v) \ , \\
T_{vv}(u,v) &= -\frac{1}{\sqrt{-\det g(u,v)}} \frac{\tau_1}{g_1 g_2 \sqrt{2}} \sqrt{1+\frac{\lambda}{u^2}} \delta(u-v) \ , \\
T_{uv}(u,v) &= -\frac{1}{\sqrt{-\det g(u,v)}} \frac{\tau_1}{g_1 g_2 \sqrt{2}} \sqrt{1+\frac{\lambda}{u^2}} \delta(u-v) \ . \qquad (5.5)
\end{aligned}$$

In these calculations we have used the fact that the factor $g'(\sigma^1)$ is present in all components of the stress energy tensor and hence we could perform the substitution $g(\sigma^1) = m$. Then the integration over m swallows up one delta function $\delta(m - u)$ so that we have replaced everywhere m with u. This implies that the second delta function becomes $\delta(u-v)$. We see from the results given above that all components of the stress energy tensor are localized around the line $u = v$. In other words the configuration (4.28) describes D1-brane that is stretched in directions u and v that are transverse to the world volume of I-brane.

Now we will calculate the components of the stress energy tensor for the solution (4.27) that is valid in the near horizon region of I-brane. As follows from (4.27) we easily get

$$H_1(\partial_1 R)^2 + H_2(\partial_1 S)^2 = \frac{\lambda_1}{R^2}(\partial_1 R)^2 + \frac{\lambda_2}{S^2}(\partial_1 S)^2 = f'^2 \ , \qquad (5.6)$$

where we have used

$$(\partial_1 R)^2 = \frac{\lambda_2}{\lambda_1(\lambda_1+\lambda_2)} f'^2 R^2 \ , \qquad (\partial_1 S)^2 = \frac{\lambda_1}{\lambda_2(\lambda_1+\lambda_2)} f'^2 S^2 \ . \qquad (5.7)$$

Then the components of the space time stress energy tensors evaluated in the near horizon approximation are equal to

$$\begin{aligned}
T_{00} &= \frac{\tau_1}{\sqrt{-\det g(x)}} \int d\sigma^1 \frac{RSf'}{g_1 g_2 \sqrt{\lambda_1\lambda_2}} \delta(R(\sigma^1) - u)\delta(S(\sigma^1) - v) \ , \\
T_{vv} &= -\frac{\tau_1}{\sqrt{-\det g(x)}} \int d\sigma^1 \frac{R\sqrt{\lambda_1\lambda_2} f'}{S(\lambda_1+\lambda_2)} \delta(R(\sigma^1) - u)\delta(S(\sigma^1) - v) \\
T_{uu} &= -\frac{\tau_1}{\sqrt{-\det g(x)}} \int d\sigma^1 \frac{S\sqrt{\lambda_1\lambda_2} f'}{R g_1 g_2(\lambda_1+\lambda_2)} \delta(R(\sigma^1) - u)\delta(S(\sigma^1) - v) \\
T_{uv} &= -\frac{\tau_1}{\sqrt{-\det g(x)}} \int d\sigma^1 \frac{\lambda_1\lambda_2 f'}{g_1 g_2(\lambda_1+\lambda_2)} \delta(R(\sigma^1) - u)\delta(S(\sigma^1) - v) \ . \qquad (5.8)
\end{aligned}$$

To evaluate T_{00} we perform the substitution $R(\sigma^1) = m$ and then we integrate over m with the result

$$\begin{aligned} T_{00} &= \frac{\tau_1}{\sqrt{-\det g}} \frac{\sqrt{\lambda_1 + \lambda_2}}{\lambda_2} S(R^{-1}(u))\delta(S(R^{-1}(u)) - v) \\ &= \frac{\tau_1}{\sqrt{-\det g}} \frac{\sqrt{\lambda_1 + \lambda_2}}{\lambda_2} u^{\frac{\lambda_1}{\lambda_2}} \delta(u^{\frac{\lambda_1}{\lambda_2}} - v) \ , \end{aligned} \tag{5.9}$$

using

$$S(R^{-1}(u)) = e^{\frac{\lambda_1}{\lambda_2} \ln u} = u^{\frac{\lambda_1}{\lambda_2}} \ . \tag{5.10}$$

In the same way we can proceed with T_{vv} and we get

$$T_{vv} = -\frac{\tau_1}{\sqrt{-\det g}} \frac{\lambda_1}{\sqrt{\lambda_1 + \lambda_2} u^{\frac{\lambda_1}{\lambda_2}}} \delta(u^{\frac{\lambda_1}{\lambda_2}} - v) \ . \tag{5.11}$$

On the other hand for T_{vv} we use the substitution $S(\sigma^1) = m$ so that we get

$$\begin{aligned} T_{uu} &= -\frac{\tau_1}{\sqrt{-\det g}} \frac{\lambda_2}{\sqrt{\lambda_1 + \lambda_2} R(S^{-1}(v))} \delta(R(S^{-1}(v)) - u) \\ &= -\frac{\tau_1}{\sqrt{-\det g}} \frac{\lambda_2}{\sqrt{\lambda_1 + \lambda_2} v^{\frac{\lambda_2}{\lambda_1}}} \delta(v^{\frac{\lambda_2}{\lambda_1}} - u) \ . \end{aligned} \tag{5.12}$$

Finally, for T_{uv} we obtain the result

$$\begin{aligned} T_{uv} &= -\frac{\tau_1}{\sqrt{-\det g}} \frac{\lambda_1 \sqrt{\lambda_1 \lambda_2}}{\sqrt{\lambda_1 + \lambda_2} R(R^{-1}(u))} \delta(S(R^{-1}(u)) - v) \\ &= -\frac{\tau_1}{\sqrt{-\det g}} \frac{\lambda_1 \sqrt{\lambda_1 \lambda_2}}{\sqrt{\lambda_1 + \lambda_2} u} \delta(u^{\frac{\lambda_1}{\lambda_2}} - v) \ . \end{aligned} \tag{5.13}$$

As follows from all components of the space time stress energy tensor the D1-brane in the near horizon limit spans the curve $u^{\lambda_1} = v^{\lambda_2}$. We also see that this result does not depend on the form of the function f given in (4.27) which can be interpreted as a consequence of the diffeomorphism invariance of D1-brane effective action.

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